JOULE-THOMSON COEFFICIENT AND SPEED OF SOUND

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Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 9, No. 1, pp. 167–168, 1968

By using relationships for the speed of sound and the Joule-Thomson coefficient it is possible to obtain some simple thermodynamic relations for the inversion curve. The inversion curve embraces a wide range of state parameters; therefore, the data obtained for this curve are of great practical value.

As is known, the Joule-Thomson coefficient is given by

$$\alpha = -\frac{T(\partial p/\partial T)_v + v(\partial p/\partial v)_T}{c_v(\partial p/\partial v)_T - T(\partial p/\partial T)_v}. \qquad (1)$$

In order to associate α with the speed of sound, we use familiar thermodynamic formulas

$$c_p - c_v = -\frac{T(\partial p/\partial T)_v^2}{(\partial p/\partial v)_T}, \qquad (2)$$

$$c^2 = -gv^2\frac{c_p}{c_v}\left(\frac{\partial p}{\partial v}\right)_T.$$

Substituting the values of the derivatives from (2) into (1) and making some simple transformations, we obtain

$$\alpha = \frac{v}{c}\left[gT\left(\frac{1}{c_v} - \frac{1}{c_p}\right)\right]^{1/2} - \frac{v}{c_p}. \qquad (3)$$

On the inversion curve $\alpha = 0$, Eq. (3) yields the following expression for the speed of sound:

$$c = \left[gT\frac{c_p}{c_v}(c_p - c_v)\right]^{1/2}. \qquad (4)$$

By comparing (4) with Laplace equation (2) we see that (4) contains thermal parameters (in particular, the derivative $(\partial p/\partial v)_T$). Therefore, (4) must be more exact than (2). Speed of sound on the inversion curve for carbon dioxide was calculated from (4) for temperatures of 669–1000° C and pressures of 92–600 bar. Calculation data was obtained from [1].

We shall give calculation results for sound velocity on the inversion curve for CO_2. The value c = 543 for the speed of sound at 1000° C was taken from [2]. This value agrees with the theoretical value to winthin 0.2%. Figure 1 shows isotherms for sound velocity based on [2]; the right sections of the isotherms shown by dashed lines are almost straight; inversion curve 1 is constructed according to outlined calculations.

Since calculations using (4) have provided us with values for sound velocity on the inversion curve and we possess data for lower pressures, we can linearly interpolate to higher pressures to obtain new data on sound velocity; however, this method of obtaining data by interpolation requires experimental verification.

As for sound velocity, we can obtain a formula for the differential adiabat:

$$K = \frac{Tc}{(\alpha + v/c_p)^2 P}\left(\frac{1}{c_v} - \frac{1}{c_p}\right). \qquad (5)$$

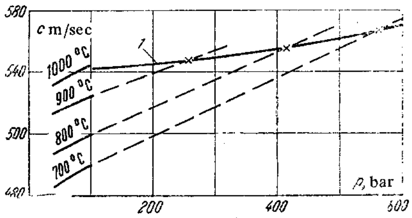

Fig. 1

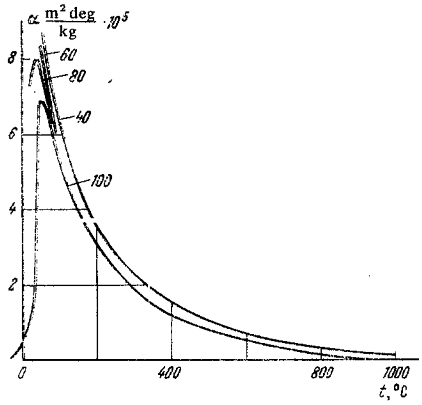

Fig. 2

On the inversion curve $\alpha = 0$, (5) takes the simple form

t =	669	700	800	900	1000	[°C],
p =	600	556	407	253	92	[bar],
c =	570	567	556	546	542 (543)	[m/sec].

Relations derived above can be used to calculate $c_p - c_v$ on the inversion curve.

By equating right sides of the second relationship in (2) and relationship (4) we obtain

$$c_p - c_v = \frac{c^2}{T}\left(\frac{\partial p}{\partial v}\right)_T. \qquad (6)$$

Equation (6) contai ns only one derivative. Usually, $c_p - c_v$ is defined by (2) as containing three derivatives. Therefore, the calculation accuracy of (6) is greater. We shall give values for $c_p - c_v$ for CO_2 on the inversion curve; these values are calculated from (6). For comparison, values for $c_p - c_v$ taken from [2] are also given.

Figure 2 shows isobars of the Joule-Thomson coefficient in α-t-coordinates as calculated from (3); two of them are supercritical (80 and 100 bar) and two are subcritical (60 and 40 bar). For the supercritical isobar and beginning with the right boundary on the inversion curve the Joule-Thomson coefficient smoothly increases from zero, has a fairly sharp maximum near the critical point, and then drops to zero (at the left side of the inversion curve). For subcritical isobars, there is a monotone increase in Joule-Thomson coefficient from zero (right boundary of inversion curve) up to values on the saturation line.

t =	700	800	900	1000	[°C]
$(c_p - c_v)_{(6)}$ =	0.2580	0.2330	0.2127	0.1962	[kJ/kg·deg]
$(c_p - c_v)_{[2]}$ =	0.2640	0.2350	0.2127	0.1959	[kJ/kg·deg]

REFERENCES

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2. N. B. Vargaftik, Handbook on Thermophysical Properties of Gases and Liquids [in Russian], Fizmatgiz, 1963.

22 June 1967 Novosibirsk